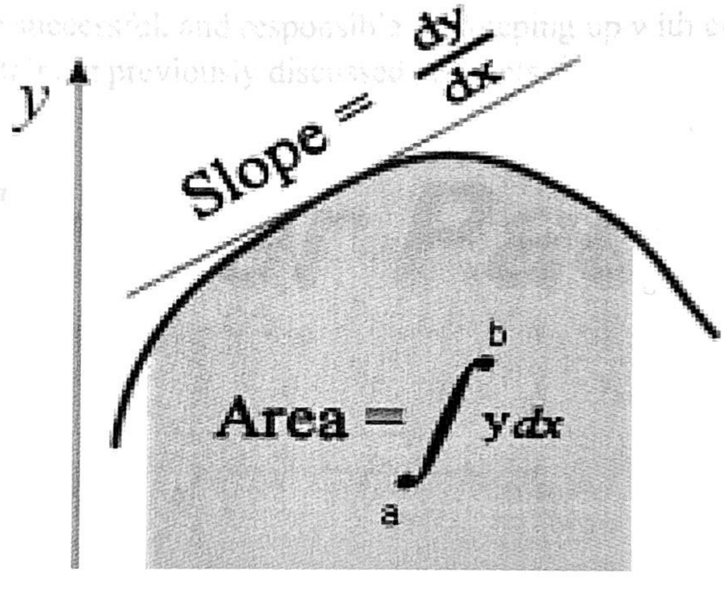
AP Calculus
Summer Packet



Congratulations and welcome to AP Calculus! While the start of next year seems far away, it is important that you have the resources and materials necessary to begin AP Calculus available to you now so that you can make sure you are ready.

The attached packet contains a course introduction, a list of previously studied topics that are essential for success in Calculus, reference sheets, and practice problems (answer keys included) on many of the concepts you are expected to know prior to starting this course. Please take the time to review the materials in this packet so that you are ready when school begins in the fall.

During the first two weeks of school there will be an in class assessment covering material from the list of previously studied topics included in this packet.

AP Calculus AB and BC, like other AP courses, are designed to help expand your knowledge and deepen your understanding of concepts at a high level. Therefore, students are expected to be dedicated to the course, committed to putting in the time necessary to be successful, and responsible for keeping up with current material while maintaining previously discussed concepts.

Enjoy your summer!

AP Calculus Introduction

Calculus is a branch of advanced mathematics that deals with problems that cannot be solved with ordinary algebra. This includes rate problems where slopes are not fixed and area and volume problems involving irregular objects. The AP Calculus courses are designed to introduce students to two main branches of calculus (differential and integral calculus) and related topics. An essential component of these courses is the continued development of critical thinking skills and problem solving skills. Therefore, students will be expected to show appropriate work and to explain results using correct calculus justifications.

Below is a list of prerequisite skills (from Algebra I, Geometry, Algebra II and PreCalculus) which are important to your success in AP Calculus. Topics with an asterisk (*) are required skills for AP Calculus BC only. In order to be prepared to meet the challenges of AP Calculus, it is recommended that students review the topics below before the start of the new school year. Students will be expected to incorporate their knowledge on these topics into calculus questions, many ***without the use of a calculator.*** Without these prerequisite skills, you will struggle to correctly solve problems next year even though you understand the calculus concepts. If you find that you are in need of a refresher on one or more of the topics below, there are some suggested online resources at the end of the list.

Following the list of prerequisite topics are several handouts with information you are expected to know and some practice problems to work on. Solutions to the practice problems are included so that you can check your work. Please note that you are responsible for ***all*** topics on this list.

Prerequisite Topics:

1. Fundamentals of Algebra
    a. Rules of exponents
    b. Factoring (including GCF, difference of two squares, trinomial, grouping, sum/difference of perfect cubes)
    c. Polynomial long and synthetic division
    d. Operations with fractions
    e. Rationalizing, simplifying and expanding expressions
    f. Simplifying complex fractions
    g. Solving equations, systems of equations and inequalities (linear and quadratic)

2. Functions
    a. Domain and range
    b. Composition of functions
    c. Operations with functions
    d. Parent functions
    e. Average rate of change
    f. Point-slope form of a line
    g. Inverse functions
    h. Odd/Even functions
    i. Piecewise functions
    j. Finding solutions with and without a calculator (including quadratic, radical, rational, absolute value, and polynomial equations)

3. Exponentials and Logarithms
    a. Rules and properties
    b. Solving equations
    c. Common and natural logarithms

4. Rational Functions
    a. Asymptotes
    b. Intercepts
    c. Holes
    d. End behavior

5. Trigonometry
    a. Right triangle trig
    b. Exact values of special angles and the Unit Circle
    c. Reciprocal trig functions
    d. Inverse trig functions
    e. Solving equations
    f. Pythagorean identities

6. Limits
    a. Techniques for evaluation (substitution, simplification, rationalization)
    b. One-sided limits
    c. Determining limits from a graph
    d. Limits as $x \to \pm\infty$
    e. Limits that approach infinity
    f. Limits that do not exist (DNE)

7. Derivatives
    a. Constant rule
    b. Power rule
    c. Product rule
    d. Quotient rule
    e. Chain rule

8. Area and Volume Formulas
    a. Area for circle, square, rectangle, triangle, equilateral triangle
    b. Volume for a cone, sphere, cylinder, rectangular prism, cube

9. Partial Fractions*

10. Sequences and Series*
    a. Identify arithmetic and geometric sequences and series
    b. Sigma notation for a series
    c. Finite and infinite series
    d. Sum of a series

11. Polar Equations*
    a. Convert between polar and rectangular coordinates/equations
    b. Graph polar equations

Online Resources:

www.khanacademy.com
www.flippedmath.com

Things to Know for Calculus

TRIGONOMETRY

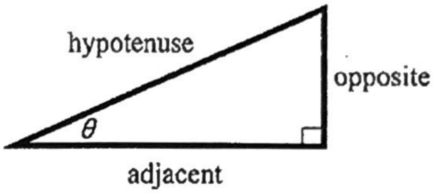
Trig Functions

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

SOH-CAH-TOA



Reciprocal Functions

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hyp}}{\text{opp}}$

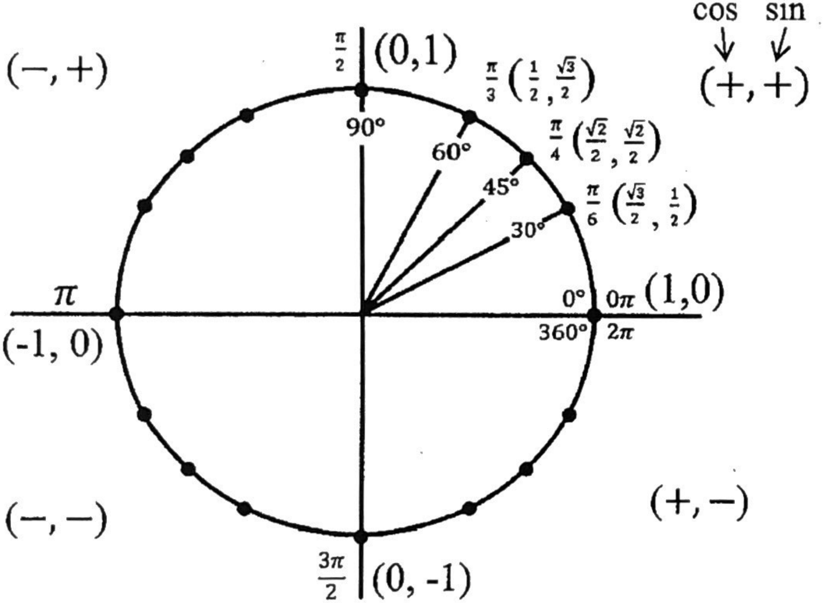
$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hyp}}{\text{adj}}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\text{adj}}{\text{opp}}$

TEST ONLY USES RADIANS!

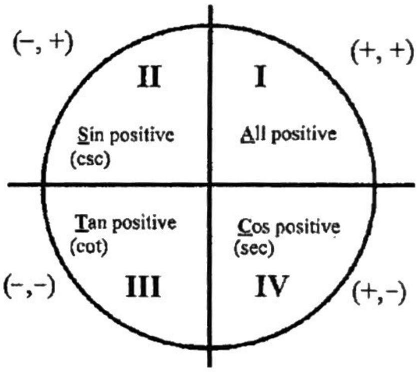
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

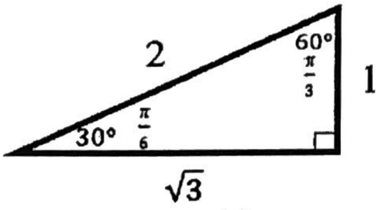
All **S**tudents **T**ake **C**alculus



SPECIAL RIGHT TRIANGLES

30° − 60° − 90° Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

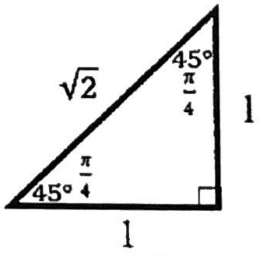


Find $\tan\left(\frac{\pi}{6}\right)$

$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ simplify to $\frac{1}{\sqrt{3}} \frac{\cdot\sqrt{3}}{\cdot\sqrt{3}} = \frac{\sqrt{3}}{3}$

45° − 45° − 90° Triangles

Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles

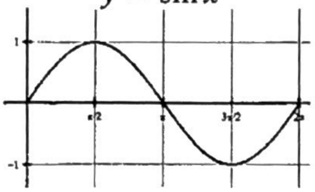


Find $\sin\left(\frac{\pi}{4}\right)$

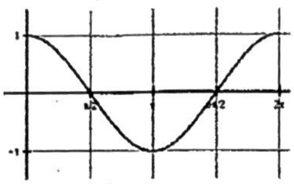
$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ simplify to $\frac{1}{\sqrt{2}} \frac{\cdot\sqrt{2}}{\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$

Graphs of trig functions

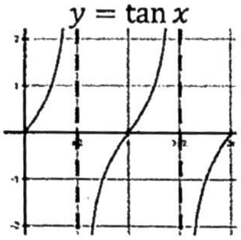
$y = \sin x$



$y = \cos x$



$y = \tan x$



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin\theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n$ or $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So… $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. $\mathbf{\sin^2 x + \cos^2 x = 1}$

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$

Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $= 2\pi r$

SURFACE AREA

Sphere $= 4\pi r^2$

LATERAL AREA

Cylinder $= 2\pi rh$

VOLUME

Sphere $= \frac{4}{3}\pi r^3$

Cylinder $= \pi r^2 h$

Cone $= \frac{1}{3}\pi r^2 h$

Prism $= Bh$

Pyramid $= \frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form
(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

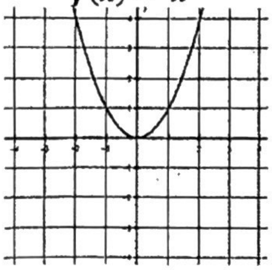
Parallel Lines
Have the same slope

Perpendicular Lines
Have the opposite reciprocal slopes

Functions

Quadratic Function

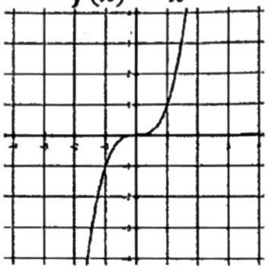
$f(x) = x^2$



$y = a(x-h)^2 + k$

Cubic Function

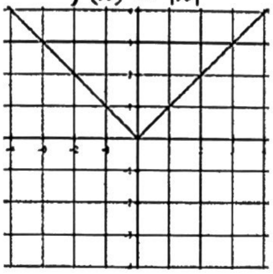
$f(x) = x^3$



$y = a(x-h)^3 + k$

Absolute Value

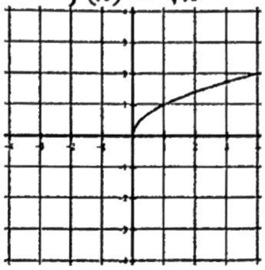
$f(x) = |x|$



$y = a|x-h| + k$

Square Root Function

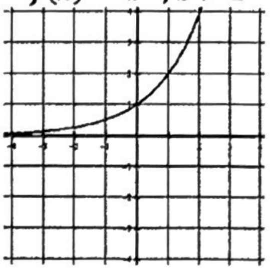
$f(x) = \sqrt{x}$



$y = a\sqrt{x-h} + k$

Exponential Function

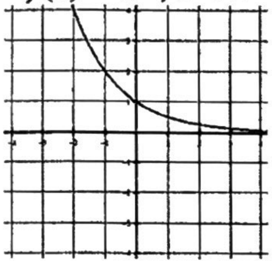
$f(x) = b^x , b > 1$



$y = a \cdot b^{(x-h)} + k$

Exponential Function

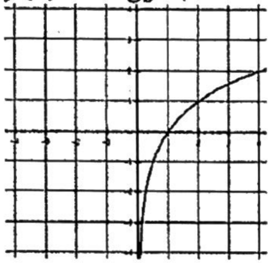
$f(x) = b^x , b < 1$



$y = a \cdot b^{(x-h)} + k$

Logarithmic Function

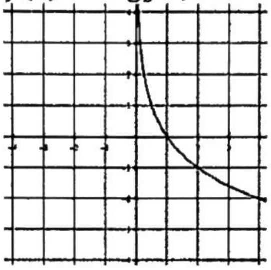
$f(x) = \log_b x , b > 1$



$y = a\log_b(x-h) + k$

Logarithmic Function

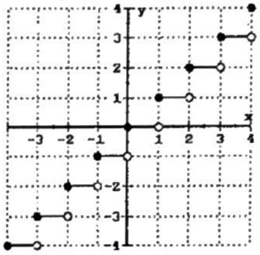
$f(x) = \log_b x , b < 1$



$y = a\log_b(x-h) + k$

Greatest Integer

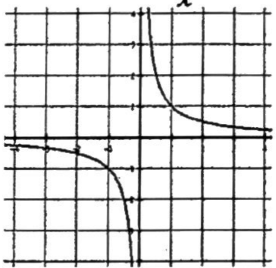
$f(x) = [\![x]\!]$



$y = a[\![x-h]\!] + k$

Rational Function

$f(x) = \dfrac{1}{x}$



$y = \dfrac{a}{x-h} + k$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4	Move down 3	Move left 2	Move right 1	Move left 2 and down 3
$y = x^2 + 4$	$y = x^2 - 3$	$y = (x+2)^2$	$y = (x-1)^2$	$y = (x+2)^2 - 3$

To flip (reflect) the function vertically $y = -x^2$

To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x-3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis ( ) versus closed [ ]

Inequality		Interval
$-3 < x \le 5$	⟷	$(-3,5]$
$-3 \le x \le 5$	⟷	$[-3,5]$
$-3 < x < 5$	⟷	$(-3,5)$
$-3 \le x < 5$	⟷	$[-3,5)$

Infinity is always open parenthesis

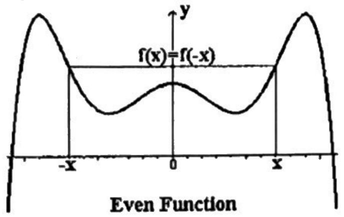
Inequality		Interval
$x < 3$	⟷	$(-\infty, 3)$
$x \le 3$ or $x > 5$	⟷	$(-\infty, 3](5, \infty)$
$x \ne 3$	⟷	$(-\infty, 3)(3, \infty)$
all Real numbers	⟷	$(-\infty, \infty)$

Even and Odd Functions

EVEN

$f(-x) = f(x)$

Symmetric about the y-axis

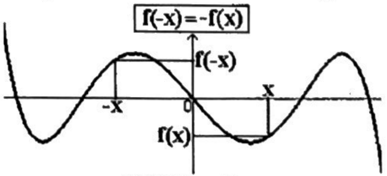


Even Function

ODD

$f(-x) = -f(x)$

Symmetric about the origin



Odd Function

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically

You can't divide by zero

You can't square root a negative

$y = \sqrt{2x+5}$

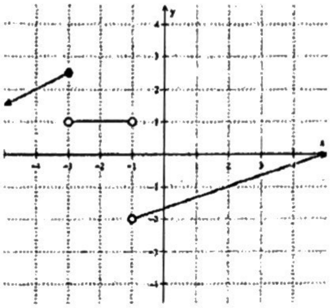
D: $[-\frac{5}{2}, \infty)$

$$y = \frac{x^2-1}{x^2+7x+12}$$

D: $(-\infty,-4)(-4,-3)(-3,\infty)$

Graphically

Just look at it



D: $(-\infty,-1)(-1,5]$

R: $(-\infty, 2.5]$

Finding zeros

Must be able to factor and use the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Special products

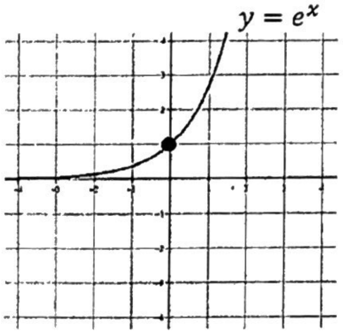
Sum of cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Exponential and Logarithmic Properties

The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logaritmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x" or "log base b of x". So…

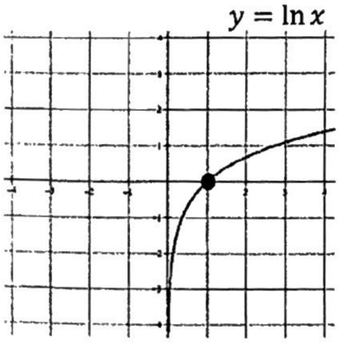
BASE e



KNOW!

$e^0=1$

Horizontal asymptote at $y=0$



KNOW!

$\ln 1 = 0$

Vertical asymptote at $x=0$

$y=\log_b x \longleftrightarrow x=b^y$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y}=b^{x-y}$	Quotient Rule	$\log_b\left(\frac{x}{y}\right)=\log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y\log_b x$
$b^{-x}=\frac{1}{b^x}$		$\log_b\left(\frac{1}{x}\right) = -\log_b x$
$b^0=1$		$\log_b 1 = 0$
$b^1=b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln e$
	Common Log	$\log_{10} x = \log x$

Calculus - SUMMER PACKET

NAME:____________________________

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

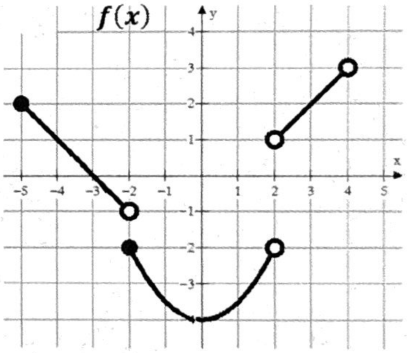
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x = ?$

$f(x) = -3$ when $x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

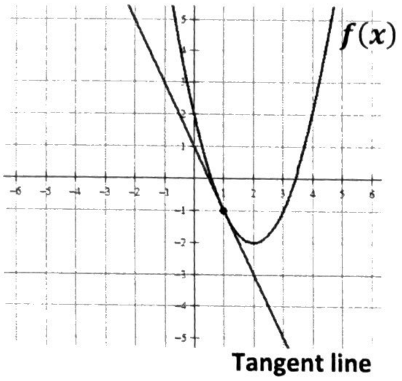
5. slope = 3 and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

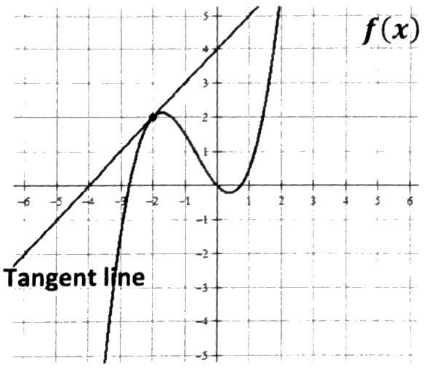
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



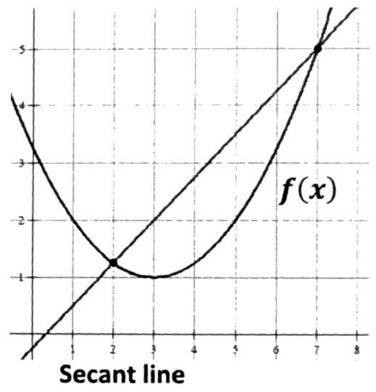
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope = $\frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

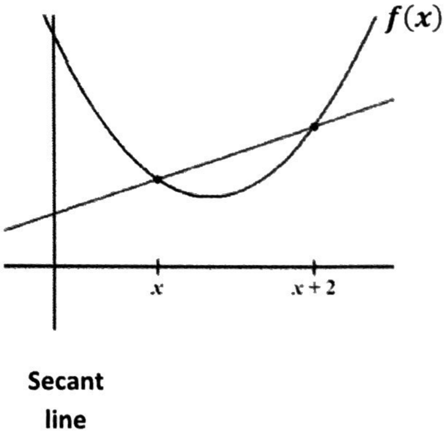
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



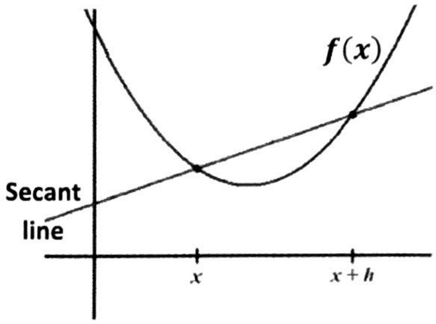
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?



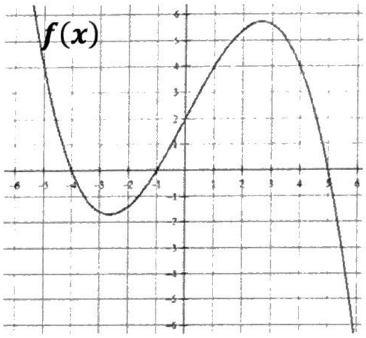
A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

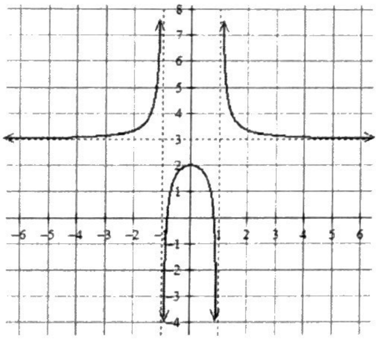
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



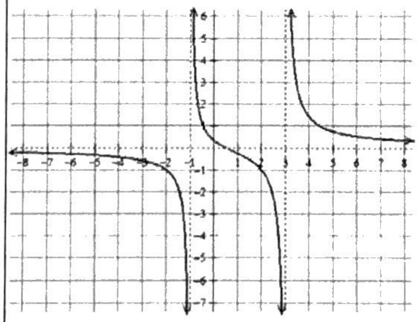
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



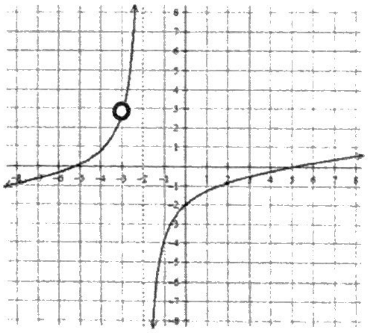
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

---

18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$

32. $\cos\frac{\pi}{4}$

33. $\sin 2\pi$

34. $\tan\pi$

35. $\sec\frac{\pi}{2}$

36. $\cos\frac{\pi}{6}$

37. $\sin\frac{\pi}{3}$

38. $\sin\frac{3\pi}{2}$

39. $\tan\frac{\pi}{4}$

40. $\csc\frac{\pi}{2}$

41. $\sin\pi$

42. $\cos\frac{\pi}{3}$

43. Find x where $0 \le x \le 2\pi$,

$$\sin x = \frac{1}{2}$$

44. Find x where $0 \le x \le 2\pi$,

$$\tan x = 0$$

45. Find x where $0 \le x \le 2\pi$,

$$\cos x = -1$$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$

47. $3e^x + 5 = 8$

48. $e^{2x} = 1$

49. $\ln x = 0$

50. $3 - \ln x = 3$

51. $\ln(3x) = 0$

52. $x^2 - 3x = 0$

53. $e^x + xe^x = 0$

54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$

56. $\cos x = -1$

57. $\cos x = \frac{\sqrt{3}}{2}$

58. $2\sin x = -1$

59. $\cos x = \frac{\sqrt{2}}{2}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

61. $\tan x = 0$

62. $\sin(2x) = 1$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$

70. $e^{\ln x}$

71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$, $h(x) = \{(3,2),(4,3),(1,6)\}$, $g(x) = \sqrt{x-3}$, $k(x) = x^2 + 5$, then determine each of the following.

81. $(f + h)(1)$

82. $(k - g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k(g(x))$

90. $g(f(2))$

Calculus - SUMMER PACKET

NAME: Solutions

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

$f(-2) = (-2)^2 - 2(-2) + 5$

$f(-2) = 4 + 4 + 5$

$\boxed{f(-2) = 13}$

2. $f(x+2) =$

$\boxed{x^2 + 2x + 5}$

3. $f(x+h) =$

$(x+h)^2 - 2(x+h) + 5$

$(x+h)(x+h) - 2x - 2h + 5$

$x^2 + xh + xh + h^2 - 2x - 2h + 5$

$\boxed{x^2 + 2xh + h^2 - 2x - 2h + 5}$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$ -4

$f(4) =$ DNE (Does not exist) or undefined

$f(-1) =$ -3.5

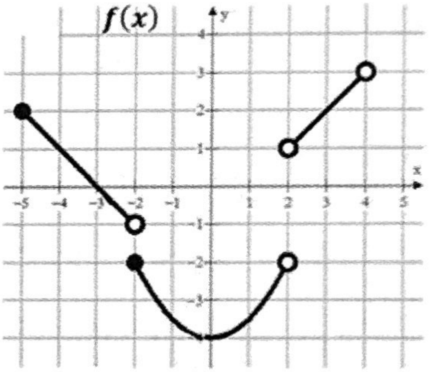
$f(-2) =$ -2

$f(2) =$ DNE (Does not exist) or undefined

$f(3) =$ 2

$f(x) = 2$ when $x = ?$ -5 and 3

$f(x) = -3$ when $x = ?$ -1.5 and 1.5



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

5. slope = 3 and (4, −2)

$y - -2 = 3(x - 4)$

$\boxed{y + 2 = 3(x - 4)}$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

$\boxed{y - 7 = -\frac{3}{2}(x + 5)}$

7. $f(4) = -8$ and $f(-3) = 12$

$m = \frac{12 - -8}{-3 - 4} = \frac{20}{-7}$

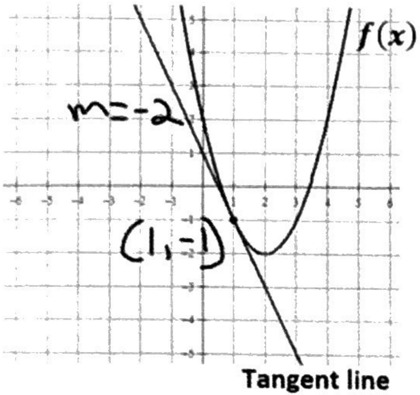
$y - 12 = -\frac{20}{7}(x + 3)$

or

$y + 8 = -\frac{20}{7}(x - 4)$

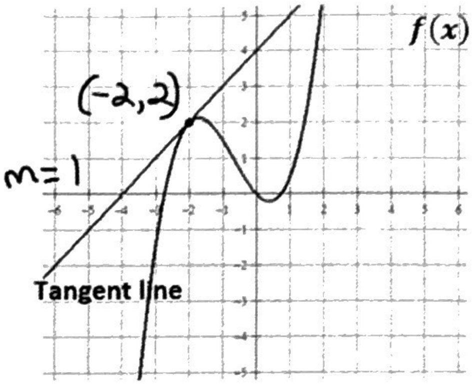
Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



$\boxed{y+1=-2(x-1)}$

9. The line tangent to $f(x)$ at $x = -2$

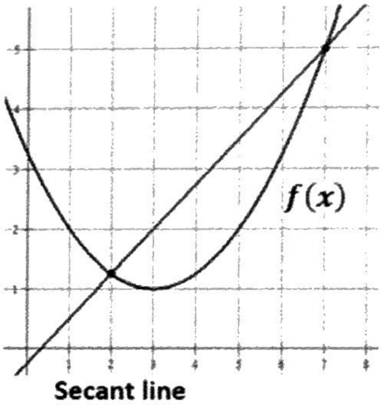


$\boxed{y-2=1(x+2)}$

MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

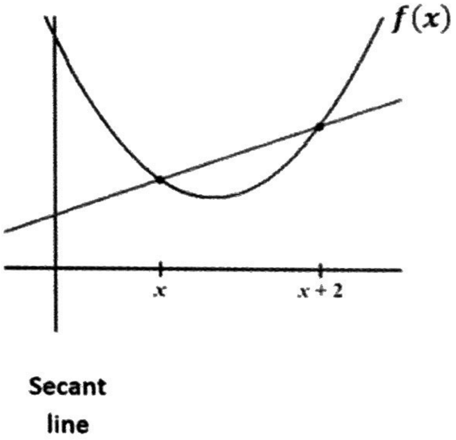
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ [✓ D) $\frac{f(7)-f(2)}{7-2}$]



11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ [✓ B) $\frac{f(x+2)-f(x)}{x+2-x}$]   C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



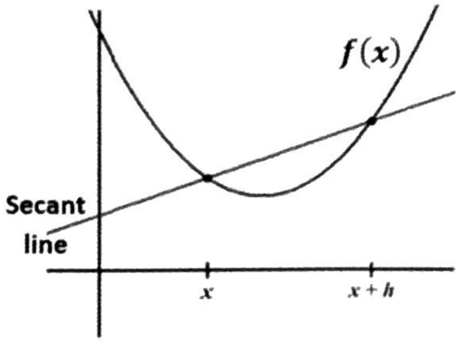
12. Which choice represents the slope of the secant line shown?

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$

B) $\frac{x-(x+h)}{f(x+h)-f(x)}$

[✓ C) $\frac{f(x+h)-f(x)}{x+h-x}$]

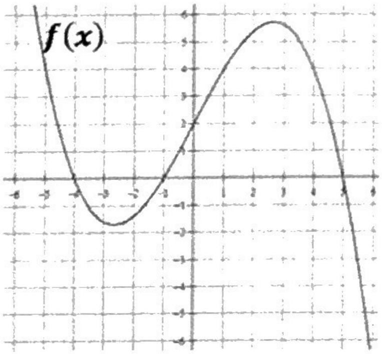
D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?

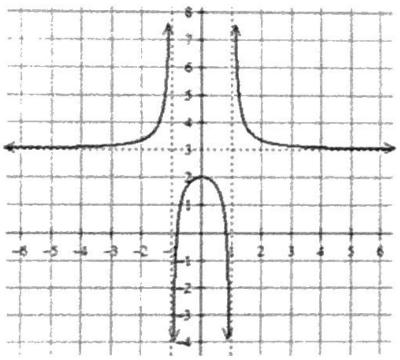
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
[✓ (E) II and III only]



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



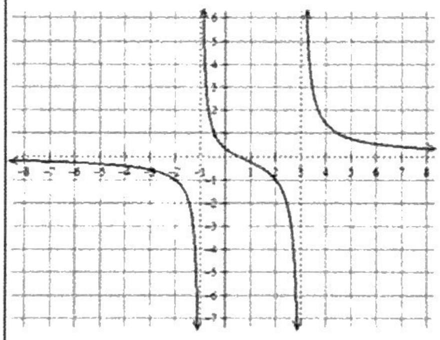
Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

Range: $(-\infty,2]\cup(3,\infty)$

Horizontal Asymptote(s): $y=3$

Vertical Asymptotes(s): $x=1$ $x=-1$

15.



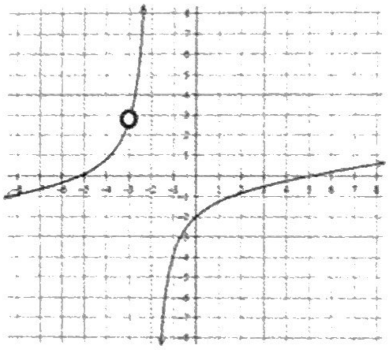
Domain: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): $y=0$

Vertical Asymptotes(s): $x=-1$ $x=3$

16.



Domain: $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): none

Vertical Asymptotes(s): $x=-2$

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at = 4 ?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

[✓ (D) $\frac{x+6}{x^2-7x+12}$]

(E) None of the above

---

18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

[✓ (D) II and III only]

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$ $x^{\frac{3}{5}} + (2x)^{\frac{1}{5}}$	20. $\sqrt{x+1}$ $(x+1)^{\frac{1}{2}}$	21. $\frac{1}{\sqrt{x+1}}$ $(x+1)^{-\frac{1}{2}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$ $x^{-\frac{1}{2}} - 2x^{-1}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$ $\frac{1}{4}x^{-3} + \frac{1}{2}x^{\frac{3}{4}}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$ $\frac{1}{4}x^{-\frac{1}{2}} - 2(x+1)^{\frac{1}{2}}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$ $\frac{1}{\sqrt{x}} - \sqrt{x^3}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$ $\frac{1}{2\sqrt{x}} + \frac{1}{x}$	27. $3x^{-\frac{1}{2}}$ $\frac{3}{\sqrt{x}}$
28. $(x+4)^{-\frac{1}{2}}$ $\frac{1}{\sqrt{x+4}}$	29. $x^{-2} + x^{\frac{1}{2}}$ $\frac{1}{x^2} + \sqrt{x}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$ $\frac{2}{x^2} + \frac{3}{2x}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$ $\frac{1}{2}$	32. $\cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$	33. $\sin 2\pi$ 0
34. $\tan\pi$ 0	35. $\sec\frac{\pi}{2}$ undefined	36. $\cos\frac{\pi}{6}$ $\frac{\sqrt{3}}{2}$
37. $\sin\frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$	38. $\sin\frac{3\pi}{2}$ -1	39. $\tan\frac{\pi}{4}$ 1
40. $\csc\frac{\pi}{2}$ 1	41. $\sin\pi$ 0	42. $\cos\frac{\pi}{3}$ $\frac{1}{2}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$ $\frac{\pi}{6}$ and $\frac{5\pi}{6}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$ 0, π, and 2π	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$ π

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$ $e^x = 1$ $\ln(e^x) = \ln(1)$ $\boxed{x=0}$	47. $3e^x + 5 = 8$ $3e^x = 3$ $e^x = 1$ $\ln e^x = \ln 1$ $\boxed{x=0}$	48. $e^{2x} = 1$ $\ln e^{2x} = \ln(1)$ $2x = 0$ $\boxed{x=0}$
49. $\ln x = 0$ $e \quad e$ $\boxed{x=1}$	50. $3 - \ln x = 3$ $-\ln x = 0$ $\ln x = 0$ $e \quad e$ $\boxed{x=1}$	51. $\ln(3x) = 0$ $e \quad e$ $3x = 1$ $\boxed{x=\frac{1}{3}}$
52. $x^2 - 3x = 0$ $x(x-3) = 0$ $\boxed{x=0 \quad x=3}$	53. $e^x + xe^x = 0$ $e^x(1+x) = 0$ $e^x = 0$ $\quad 1+x = 0$ not possible $\boxed{x=-1}$	54. $e^{2x} - e^x = 0$ $e^x(e^x-1) = 0$ $e^x = 0$ $\quad e^x - 1 = 0$ not possible $\quad e^x = 1$ $\boxed{x=0}$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

56. $\cos x = -1$

$x = \pi$

57. $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

58. $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}}$

59. $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$ and $\frac{7\pi}{4}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{6}$ $\frac{x}{2} = \frac{11\pi}{6}$

$\boxed{x = \frac{\pi}{3}}$ $x = \frac{11\pi}{3}$

not in the domain interval

61. $\tan x = 0$

$\frac{\sin x}{\cos x} = 0 \rightarrow \sin x = 0$

$\boxed{x = 0, \pi, 2\pi}$

62. $\sin(2x) = 1$

$2x = \frac{\pi}{2}$ and $2x = \frac{5\pi}{2}$

$\boxed{x = \frac{\pi}{4} \text{ and } x = \frac{5\pi}{4}}$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{4} = \frac{\pi}{3}$ $\frac{x}{4} = \frac{2\pi}{3}$

$\boxed{x = \frac{4\pi}{3}}$ $x = \frac{8\pi}{3}$

not in the domain

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$	$x \ge 4$	$y \ge 0$
65. $y = (x-3)^2$	$\mathbb{R}$ all real numbers	$y \ge 0$
66. $y = \ln x$	$x > 0$	$\mathbb{R}$
67. $y = e^x$	$\mathbb{R}$	$y > 0$
68. $y = \sqrt{4-x^2}$	$-2 \le x \le 2$	$0 \le y \le 2$

Simplify.

69. $\frac{\sqrt{x}}{x}$

$x^{\frac{1}{2}-1}$

$x^{-\frac{1}{2}}$

$\boxed{\frac{1}{\sqrt{x}}}$

70. $e^{\ln x}$

$\boxed{x}$

71. $e^{1+\ln x}$

$e^1 \cdot e^{\ln x}$

$\boxed{ex}$

72. $\ln 1$

$\boxed{0}$

73. $\ln e^7$

$\boxed{7}$

74. $\log_3 \frac{1}{3}$

$\log_3 3^{-1}$

$\boxed{-1}$

75. $\log_{1/2} 8$

$\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-3}$

$\boxed{-3}$

76. $\ln\frac{1}{2}$ Calculator needed

$\approx \boxed{-0.693}$

77. $27^{\frac{2}{3}}$

$\sqrt[3]{27}^{\,2}$

3^2

$\boxed{9}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

$20a^{\frac{2}{3}+\frac{3}{2}}$

$\boxed{20a^{\frac{13}{6}}}$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

$\frac{1}{3}x^{1--\frac{1}{3}}\ y^{-2--5}$

$\boxed{\frac{1}{3}x^{\frac{4}{3}}y^{3}}$

80. $\left(4a^{5/3}\right)^{3/2}$

$\sqrt{4}^{\,3}\ a^{\frac{5}{3}\cdot\frac{3}{2}}$

$\boxed{8a^{\frac{5}{2}}}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$, $h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2+5$, **then determine each of the following.**

81. $(f+h)(1)$

$f(1)+h(1)$

$7 + 6$

$\boxed{13}$

82. $(k-g)(5)$

$k(5)-g(5)$

$(25+5)-(\sqrt{2})$

$\boxed{30-\sqrt{2}}$

83. $f(h(3))$

$f(2)$

$\boxed{4}$

84. $g(k(7))$

$g(7^2+5)$

$g(54)$

$\boxed{\sqrt{54-3} = \sqrt{51}}$

85. $h(3)$

$\boxed{2}$

86. $g(g(9))$

$g(\sqrt{6})$

$\boxed{\sqrt{\sqrt{6}-3}}$

87. $f^{-1}(4)$

$\boxed{2}$

88. $k^{-1}(x)$

$x = y^2+5$

$x-5 = y^2$

$\boxed{y = -\sqrt{x-5}}$

89. $k(g(x)) = (\sqrt{x-3})^2 + 5$

$x-3+5$

$\boxed{x+2}$

90. $g(f(2))$

$g(4) = \sqrt{4-3}$

$\boxed{1}$

Name: ______________________________ Date:______________ Period: _____ **Review**

1 Review – Limits

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points.

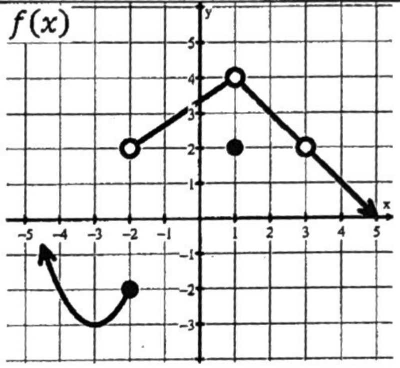
1.1 Limits Graphically:

What is a limit?

The **y-value** a function approaches at a given x-value.

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined.*"

1. $\lim_{x\to 3} f(x) =$
2. $\lim_{x\to 1} f(x) =$
3. $f(3) =$
4. $f(-2) =$
5. $\lim_{x\to 2} f(x) =$
6. $\lim_{x\to -2^+} f(x) =$
7. $f(1) =$
8. $\lim_{x\to -2^-} f(x) =$



1.2 Limits Analytically:

Finding a limit:

1. Direct Substitution.
2. Simplify and then try direct substitution.
    - a. Factor and Cancel.
    - b. Rationalize if you see square roots.

Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x} = \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x} =$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x - 1}{x} =$$

← We will cover these during the firstweek of school.

Evaluate each limit.

9. $\lim_{x\to -4}(2x^2+3x-2)$	10. $\lim_{x\to 1}\sqrt{7x+42}$	11. $\lim_{x\to 13} 2$	12. $\lim_{x\to 10}\frac{x^2-5x-50}{x-10}$

13. $\lim\limits_{x \to 0} \frac{\sqrt{x+19}-\sqrt{19}}{x}$

14. $\lim\limits_{x \to 0} \frac{\frac{1}{x+1}-1}{x}$

15. $\lim\limits_{x \to 0} \frac{\sin(7x)}{11x}$

16. $\lim\limits_{x \to 0} \frac{\sin^2(3x)}{\sin^2(5x)}$

1.3 Asymptotes:

Vertical Asymptotes:

If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach $-\infty$ or ∞.

Horizontal asymptotes:

$\lim\limits_{x \to \infty} \frac{f(x)}{g(x)}$ will produce a horizontal asymptote at

- $y = 0$ if g increases faster than f.
- $y = \frac{a}{b}$ if g and f are increasing at the relative same amount where a and b are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Evaluate each limit.			Find all horizontal asymptotes.
17. $\lim\limits_{x \to \infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$	18. $\lim\limits_{x \to \infty} x^5 3^{-x}$	19. $\lim\limits_{x \to \infty} \sin \frac{x+3\pi x^2}{2x^2}$	20. $f(x) = \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$

1.4 Continuity:

Types of Discontinuities:

1. Removable (hole).
2. Discontinuity due to vertical asymptote.
3. Jump discontinuity.

Finding Domain:

Restrictions occur with two scenarios:

1. Denominators can't be zero.
2. Even radicals can't be negative.

Name: Solutions Date:____________ Period: _____ **Review**

1 Review – Limits

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points.

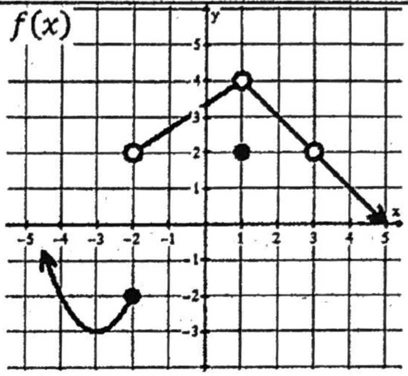
1.1 Limits Graphically:

What is a limit?

The **y-value** a function approaches at a given x-value.

Give the value of each statement. If the value does not exist, write *"does not exist"* or *"undefined."*

1. $\lim_{x\to 3} f(x) = 2$
2. $\lim_{x\to 1} f(x) = 4$
3. $f(3) =$ DNE
4. $f(-2) = -2$
5. $\lim_{x\to 2} f(x) = 3$
6. $\lim_{x\to -2^+} f(x) = 2$
7. $f(1) = 2$
8. $\lim_{x\to -2^-} f(x) = -2$



1.2 Limits Analytically:

Finding a limit:

1. Direct Substitution.
2. Simplify and then try direct substitution.
   a. Factor and Cancel.
   b. Rationalize if you see square roots.
3. L'Hôpital's Rule (for indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$)

Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0 \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

Evaluate each limit.

9. $\lim_{x\to -4}(2x^2+3x-2)$

$2(-4)^2+3(-4)-2$

$32-12-2$

$\boxed{18}$

10. $\lim_{x\to 1}\sqrt{7x+42}$

$\sqrt{7+42}$

$\sqrt{49}$

$\boxed{7}$

11. $\lim_{x\to 13} 2$

$\boxed{2}$

12. $\lim_{x\to 10}\frac{x^2-5x-50}{x-10}$

$\frac{\cancel{(x-10)}(x+5)}{\cancel{x-10}}$

$10+5 = \boxed{15}$

13. $\lim\limits_{x\to 0}\frac{\sqrt{x+19}-\sqrt{19}}{x}\cdot\frac{\sqrt{x+19}+\sqrt{19}}{\sqrt{x+19}+\sqrt{19}}=\lim\limits_{x\to 0}\frac{(x+19)-(19)}{x(\sqrt{x+19}+\sqrt{19})}$

$\lim\limits_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{x+19}+\sqrt{19})}=\frac{1}{\sqrt{0+19}+\sqrt{19}}=\frac{1}{2\sqrt{19}}=\boxed{\frac{\sqrt{19}}{38}}$

14. $\lim\limits_{x\to 0}\frac{\frac{1}{x+1}-1}{x}\cdot\frac{x+1}{x+1}=\lim\limits_{x\to 0}\frac{1-(x+1)}{x(x+1)}$

$\lim\limits_{x\to 0}\frac{-\cancel{x}}{\cancel{x}(x+1)}=\lim\limits_{x\to 0}\frac{-1}{0+1}=\boxed{-1}$

15. $\lim\limits_{x\to 0}\frac{\sin(7x)}{11x}=\lim\limits_{x\to 0}\frac{1}{11}\cdot\frac{\sin(7x)}{x}\cdot\frac{7}{7}$

$\lim\limits_{x\to 0}\frac{7}{11}\cdot\frac{\sin(7x)}{7x}=\boxed{\frac{7}{11}}$

16. $\lim\limits_{x\to 0}\frac{\sin^2(3x)}{\sin^2(5x)}$

$\lim\limits_{x\to 0}\frac{\sin(3x)}{3x}\frac{\sin(3x)}{3x}\frac{5x}{\sin(5x)}\frac{5x}{\sin(5x)}\cdot\frac{3}{5}\frac{3}{5}=\boxed{\frac{9}{25}}$

1.3 Asymptotes:

Vertical Asymptotes:

If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach $-\infty$ or ∞.

Horizontal asymptotes:

$\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}$ will produce a horizontal asymptote at

- $y=0$ if g increases faster than f.
- $y=\frac{a}{b}$ if g and f are increasing at the relative same amount where a and b are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Evaluate each limit.			Find all horizontal asymptotes.
17. $\lim\limits_{x\to\infty}\frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$ $\frac{4}{2}=\boxed{2}$	18. $\lim\limits_{x\to\infty}x^5 3^{-x}$ $\lim\limits_{x\to\infty}\frac{x^5}{3^x}$ BIG! $\boxed{0}$	19. $\lim\limits_{x\to\infty}\sin\frac{x+3\pi x^2}{2x^2}$ $\sin\left(\frac{3\pi}{2}\right)$ $\boxed{-1}$	20. $f(x)=\frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$ as $x\to\infty$, $y=4/5$ as $x\to-\infty$, $y=-4/5$

1.4 Continuity:

Types of Discontinuities:

1. Removable (hole).
2. Discontinuity due to vertical asymptote.
3. Jump discontinuity.

Finding Domain:

Restrictions occur with two scenarios:

1. Denominators can't be zero.
2. Even radicals can't be negative.

Name: ______________________________ Date: ______________ Period: _____ **Review**

2 Review – The Derivative

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 2.

2.1 Average Rate of Change

A continuous function $f(x)$ on the interval $[\boldsymbol{a}, \boldsymbol{b}]$ has an average rate of change of

$$\frac{f(b)-f(a)}{b-a} \quad \text{or} \quad \frac{f(a)-f(b)}{a-b}$$

This is also the **SLOPE** of the **SECANT** line.

Find the average rate of change for each function on the given interval. Use units when necessary.

1. $w(t) = 5t^2 - 5t + 1; [-2, 1]$
2. $s(x) = \frac{x+5}{3}$; $[1, 7]$
3. $B(t) = \cos\left(\frac{\pi}{3}t\right)$; $\left[\frac{3}{2}, 6\right]$
   B represents wild boar
   t represents weeks

2.2 Definition of the Derivative

Defintion of the derivative:

This limit gives an expression that calculates the *instantaneous* rate of change (slope of the tangent line) of $f(x)$ at any given x-value.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative at a point:

Finding the derivative at a specific x-value. We will call this value c.

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

or

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Find the derivative using limits. WRITE SMALL!!

4. $y = 2x^2 + 3x - 1$

Create an equation of the tangent line of f at the given point. Leave in point-slope.

5. $f(x) = -2x^3 + 3x;$
$f'(x) = -6x^2 + 3; \;\; x = -1$

Identify the original function $f(x)$, and what value of c to evaluate $f'(c)$.

6. $\lim_{h \to 0} \frac{-(3+h)^2+(3+h)-4+(10)}{h}$

$f(x) =$ ______________________

$c =$ __________

7. $\lim_{x \to 5} \frac{(4x-2x^3)+(230)}{x-5}$

$f(x) =$ ______________________

$c =$ __________

2.3 Differentiability

8. When does the derivative fail to exist?

9. What is the difference between the Mean Value Theorem and the Intermediate Value Theorem?

Given $f(x)$ and $f'(x)$ on a given interval $[a, b]$, find a value c that satisfies the Mean Value Theorem.

10. $f(x) = 4x^2 - 3x + 5; \;\; [-2, 2]$
$f'(x) = 8x - 3$

Using a calculator find the value of the derivative at a given point.

11. $f(x) = 0.2 \ln x$

$f'(0.7) =$

Check your 2.3 packet on matching graphs between f and f'.

Name: Solutions Date:__________ Period: ____ **Review**

2 Review – The Derivative

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 2.

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A continuous function $f(x)$ on the interval $[\boldsymbol{a}, \boldsymbol{b}]$ has an average rate of change of

$$\frac{f(b)-f(a)}{b-a} \quad \text{or} \quad \frac{f(a)-f(b)}{a-b}$$

This is also the **SLOPE** of the **TANGENT** line.

Find the average rate of change for each function on the given interval. Use units when necessary.

1. $w(t) = 5t^2 - 5t + 1; [-2, 1]$

$w(-2) = 31$

$w(1) = 1$

$$\frac{31-1}{-2-1} = \frac{30}{-3} =$$

$$\boxed{-10}$$

2. $s(x) = \frac{x+5}{3}$; $[1, 7]$

$S(1) = 2$

$S(7) = 4$

$$\frac{2-4}{1-7} = \frac{-2}{-6}$$

$$\boxed{1/3}$$

3. $B(t) = \cos\left(\frac{\pi}{3}t\right)$; $\left|\frac{3}{2}, 6\right|$

B represents wild boar

t represents weeks

$B\left(\frac{3}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$B(6) = \cos(2\pi) = 1$

$$\frac{0-1}{3/2-6} = \frac{-1}{-9/2} = 2/9$$

$$\boxed{2/9 \text{ boar per week}}$$

2.2 Definition of the Derivative

Defintion of the derivative:

This limit gives an expression that calculates the *instantaneous* rate of change (slope of the tangent line) of $f(x)$ at any given x-value.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative at a point:

Finding the derivative at a specific x-value. We will call this value c.

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

or

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Find the derivative using limits. WRITE SMALL!!

4. $y = 2x^2 + 3x - 1$

$$y' = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) - 1 - [2x^2+3x-1]}{h}$$

$$\frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h\cancel{-1}-\cancel{2x^2}-\cancel{3x}\cancel{+1}}{h}$$

$$y' = \lim_{h\to 0} \frac{h(4x+2h+3)}{h}$$

$$\boxed{y' = 4x+3}$$

Create an equation of the tangent line of f at the given point. Leave in point-slope.

5. $f(x) = -2x^3 + 3x;$
$f'(x) = -6x^2 + 3;\ \ x = -1$

$f(-1) = -1$

$f'(-1) = -3$

$$\boxed{y+1 = -3(x+1)}$$

Identify the original function $f(x)$, and what value of c to evaluate $f'(c)$.

6. $\lim\limits_{h\to 0} \frac{-(3+h)^2+(3+h)-4+(10)}{h}$

$f(x) =$ $-x^2+x-4$

$c =$ 3

7. $\lim\limits_{x\to 5} \frac{(4x-2x^3)+(130)}{x-5}$

$f(x) =$ $4x-2x^3$

$c =$ 5

2.3 Differentiability

8. When does the derivative fail to exist?

Discontinuity, corner or cusp, vertical tangent.

9. What is the difference between the Mean Value Theorem and the Intermediate Value Theorem?

The MVT states focuses on the rate of change (slope) of the function, while the IVT focuses on the value (y-value) of the function.

Given $f(x)$ and $f'(x)$ on a given interval $[a,b]$, find a value c that satisfies the Mean Value Theorem.

10. $f(x) = 4x^2 - 3x + 5;\ [-2, 2]$
$f'(x) = 8x - 3$

$f(-2) = 27$

$f(2) = 15$

$$\frac{27-15}{-2-2} = -3$$

$8x - 3 = -3$

$$\boxed{x = 0}$$

Using a calculator find the value of the derivative at a given point.

11. $f(x) = 0.2 \ln x$
$f'(0.7) =$

$$\boxed{0.2857}$$

Check your 2.3 packet on matching graphs between f and f'.

AP Calculus

Name________________________

Constant and Power Rule Notes

Constant & Power Rules

Constant Rule:

The derivative of $f(x) = c$ (where c is a constant) is $f'(x) = 0$.

Power Rule:

The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$ (It may be necessary to rewrite some functions in order to apply the power rule more easily, use the algebra you know to help.)

1. $y = x^9$

5. $f(x) = \dfrac{1}{x^4}$

2. $y = -3$

6. $y = \sqrt[4]{x^3}$

3. $y = 2x^{12}$

7. $y = \sqrt{x}$

4. $f(x) = \pi$

8. $f(x) = \dfrac{4}{x^2}$

AP Calculus Name Solutions

Constant and Power Rule Notes

Constant & Power Rules

Constant Rule:

The derivative of $f(x) = c$ (where c is a constant) is $f'(x) = 0$.

Power Rule:

The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$ (It may be necessary to rewrite some functions in order to apply the power rule more easily, use the algebra you know to help.)

1. $y = x^9$

$\frac{dy}{dx} = 9x^8$

2. $y = -3$

$\frac{dy}{dx} = 0$

3. $y = 2x^{12}$

$\frac{dy}{dx} = 24x^{11}$

4. $f(x) = \pi$

$f'(x) = 0$

5. $f(x) = \frac{1}{x^4} \rightarrow$ rewrite $f(x) = x^{-4}$

$f'(x) = -4x^{-3}$ or $\frac{-4}{x^3}$

6. $y = \sqrt[4]{x^3} \rightarrow$ rewrite $y = x^{3/4}$

$\frac{dy}{dx} = \frac{3}{4}x^{-1/4}$ or $\frac{3}{4\sqrt[4]{x}}$

7. $y = \sqrt{x} \rightarrow$ rewrite $y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$ or $\frac{1}{2\sqrt{x}}$

8. $f(x) = \frac{4}{x^2} \rightarrow$ rewrite $f(x) = 4x^{-2}$

$f'(x) = -8x^{-3}$

AP Calculus
Product and Quotient Rule Notes

Name ______________________________

Product & Quotient Rules

Product Rule:

The derivative of $f(x)g(x)$ is $f'(x)g(x)+f(x)g'(x)$

1. $f(x)=(2x^3)(5x+1)$

2. $f(x)=2x^{1/2}(3x-1)$

3. $f(x)=(\frac{2}{3}x^3+6x-2)(4\sqrt{x}-6)$

Quotient Rule:

The derivative of $\frac{f(x)}{g(x)}$ is $\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

1. $f(x)=\frac{x^2+3}{x^2-4}$

2. $f(x)=\frac{x+1}{x^2+2x+2}$

3. $f(x)=\left(\frac{x+4}{x+3}\right)(2x+5)$

4. $f(x)=\frac{x^2+c^2}{x^2-c^2}$ * *c* is a constant

AP Calculus
Product and Quotient Rule Notes

Name Solutions

Product & Quotient Rules

Product Rule:

The derivative of $f(x)g(x)$ is $f^{'}(x)g(x)+f(x)g^{'}(x)$

1. $f(x)=(2x^3)(5x+1)$

$$f'(x)=(2x^3)(5)+(5x+1)(6x^2)$$

$$f'(x)=10x^3+30x^3+6x^2$$

$$f'(x)=40x^3+6x^2$$

2. $f(x)=2x^{\frac{1}{2}}(3x-1)$

$$f'(x)=2x^{1/2}(3)-(3x-1)(x^{-1/2})$$

$$f'(x)=6x^{1/2}-3x^{1/2}+x^{-1/2}$$

$$f'(x)=3x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}} \quad \text{or} \quad 3\sqrt{x}+\frac{1}{\sqrt{x}}$$

3. $f(x)=(\frac{2}{3}x^3+6x-2)(4\sqrt{x}-6)$ $\rightarrow (4x^{\frac{1}{2}}-6)$

$$f'(x)=(\tfrac{2}{3}x^3+6x-2)(2x^{-1/2})+(4x^{\frac{1}{2}}-6)(2x^2+6)$$

$$f'(x)=\frac{4}{3}x^{5/2}+12x^{1/2}-4x^{-1/2}+8x^{5/2}+24x^{1/2}-12x^2-36$$

$$f'(x)=\frac{28}{3}x^{5/2}-12x^2+36x^{\frac{1}{2}}-4x^{-1/2}-36$$

Quotient Rule:

The derivative of $\frac{f(x)}{g(x)}$ is $\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

1. $f(x)=\frac{x^2+3}{x^2-4}$

$$f'(x)=\frac{(x^2-4)(2x)-(x^2+3)(2x)}{(x^2-4)^2}=\frac{2x^3-8x-2x^3-6x}{(x^2-4)^2}$$

$$f'(x)=\frac{-14x}{(x^2-4)^2}$$

2. $f(x)=\frac{x+1}{x^2+2x+2}$

$$f'(x)=\frac{(x^2+2x+2)(1)-\overset{(2x^2+4x+2)}{(x+1)(2x+2)}}{(x^2+2x+2)^2}=\frac{x^2+2x+2-2x^2-4x-2}{(x^2+2x+2)^2}$$

$$f'(x)=\frac{-x^2-2x}{(x^2+2x+2)^2}$$

3. $f(x)=\left(\frac{x+4}{x+3}\right)(2x+5) \longrightarrow$ rewrite $\frac{(x+4)(2x+5)}{x+3}=\frac{2x^2+13x+20}{x+3}$

$$f'(x)=\frac{(x+3)(4x+13)-(2x^2+13x+20)(1)}{(x+3)^2}=\frac{4x^2+25x+39-2x^2-13x-20}{(x+3)^2}$$

$$f'(x)=\frac{2x^2+12x+19}{(x+3)^2}$$

4. $f(x)=\frac{x^2+c^2}{x^2-c^2}$ * c is a constant

$$f'(x)=\frac{(x^2-c^2)(2x)-(x^2+c^2)(2x)}{(x^2-c^2)^2}=\frac{2x^3-2xc^2-2x^3-2xc^2}{(x^2-c^2)^2}$$

$$f'(x)=\frac{-4xc^2}{(x^2-c^2)^2}$$

AP Calculus
Chain Rule Notes

Name ______________________

Chain Rule

The derivative of $f(g(x))$ is $f'(g(x)) * g'(x)$

1. $y=\sqrt{x^3+7}$

2. $y=\dfrac{2}{(5x+1)^2}$

3. $y=\left(5x^2-3x\right)^8$

4. $y=(x^2+5)^3(x^3-1)^4$

5. $f(x)=\sqrt{5x^9}$

6. $y=(9-3x)^{-4}$

7. $f(x)=\dfrac{5}{\left(x^2+3x+5\right)^3}$

8. $y=\dfrac{1}{\sqrt[3]{(2x+4)^5}}$

AP Calculus
Chain Rule Notes

Name Solutions

Chain Rule

The derivative of $f(g(x))$ is $f'(g(x))*g'(x)$

1. $y=\sqrt{x^3+7} \rightarrow y=(x^3+7)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(x^3+7)^{-1/2} \cdot 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2\sqrt{x^3+7}}$$

2. $y=\frac{2}{(5x+1)^2} \rightarrow 2(5x+1)^{-2}$

$$\frac{dy}{dx} = -4(5x+1)^{-3}(5)$$

$$\frac{dy}{dx} = \frac{-20}{(5x+1)^3}$$

3. $y=\left(5x^2-3x\right)^8$

$$\frac{dy}{dx} = 8(5x^2-3x)^7(10x-3)$$

$$\frac{dy}{dx} = 8(10x-3)(5x^2-3x)^7$$

4. $y=(x^2+5)^3(x^3-1)^4$ * Product + chain rule

$$\frac{dy}{dx} = (x^2+5)^3 \cdot 4(x^3-1)^3(3x^2) + (x^3-1)^4 \cdot 3(x^2+5)^2(2x)$$

$$\frac{dy}{dx} = 12x^2(x^2+5)^3(x^3-1)^3 + 6x(x^3-1)^4(x^2+5)^2$$

5. $f(x)=\sqrt{5x^9} \rightarrow (5x^9)^{1/2}$

$$f'(x) = \frac{1}{2}(5x^9)^{-1/2}(45x^8)$$

$$f'(x) = \frac{45x^8}{2\sqrt{5x^9}}$$

6. $y=(9-3x)^{-4}$

$$\frac{dy}{dx} = -4(9-3x)^{-5}(-3)$$

$$\frac{dy}{dx} = \frac{12}{(9-3x)^5}$$

7. $f(x)=\frac{5}{\left(x^2+3x+5\right)^3} \rightarrow 5(x^2+3x+5)^{-3}$

$$f'(x) = -15(x^2+3x+5)^{-4}(2x+3)$$

$$f'(x) = \frac{-15(2x+3)}{(x^2+3x+5)^4}$$

8. $y=\frac{1}{\sqrt[3]{(2x+4)^5}} \rightarrow (2x+4)^{\frac{5}{3}}$

$$\frac{dy}{dx} = \frac{5}{3}(2x+4)^{2/3}(2)$$

$$\frac{dy}{dx} = \frac{10}{3}(2x+4)^{2/3}$$

AP Calculus

Name______________________________

Using Derivatives

Sum and Difference with Derivatives:

If $h(x) = f(x) \pm g(x)$ then $h'(x) = f'(x) \pm g'(x)$

Find $\frac{dy}{dx}$:

1. $y = 5x^3 - 2x^2 + 3x + 5$

2. $y = 9\sqrt{x} - \frac{1}{x^2}$

Slope of a tangent line to a graph:

The slope of the tangent line to a function at $x = a$ is found by evaluating $f'(a)$.

Find the slope of the tangent line for:

1. $y = 2x^2 - 3x$ at $x = 2$

2. $y = \sqrt{x-1}$ at $(5, 2)$

Equation of the tangent line to a graph at a point:

At the point (a,b) the equation of the tangent line (in point-slope form) is $y-b=f'(a)(x-a)$

Write an equation for the tangent line to the function at the given point in point-slope form:

1. $f(x)=x^2+5x-3 \quad (-1,-7)$

2. $f(x)=\sqrt{x+3} \quad (6,3)$

3. $f(x)=3x^2(x+2)^2 \quad (1,27)$

4. $f(x)=\frac{1}{\sqrt{x}} \quad \left(4,\frac{1}{2}\right)$

AP Calculus

Name Solutions

Using Derivatives

Sum and Difference with Derivatives:

If $h(x) = f(x) \pm g(x)$ then $h'(x) = f'(x) \pm g'(x)$

Find $\frac{dy}{dx}$:

1. $y = 5x^3 - 2x^2 + 3x + 5$

$\frac{dy}{dx} = 15x^2 - 4x + 3$

2. $y = 9\sqrt{x} - \frac{1}{x^2}$ → rewrite $y = 9x^{\frac{1}{2}} - x^{-2}$

$\frac{dy}{dx} = \frac{9}{2}x^{-1/2} + 2x^{-3}$

or $\frac{dy}{dx} = \frac{9}{2\sqrt{x}} + \frac{2}{x^3}$

Slope of a tangent line to a graph:

The slope of the tangent line to a function at $x = a$ is found by evaluating $f'(a)$.

Find the slope of the tangent line for:

1. $y = 2x^2 - 3x$ at $x = 2$

$\frac{dy}{dx} = 4x - 3$ at $x = 2$

$4(2) - 3 = 5$

slope of tangent line at $x = 2$ is $\boxed{5}$

2. $y = \sqrt{x-1}$ at $(5, 2)$

$y = (x-1)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x-1)^{-1/2}$ or $\frac{1}{2\sqrt{x-1}}$ at $x = 5$

$\frac{1}{2\sqrt{5-1}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

Slope of tangent line at $x = 5$ is $\boxed{\frac{1}{4}}$

Equation of the tangent line to a graph at a point:

At the point (a,b) the equation of the tangent line (in point-slope form) is $y-b=f'(a)(x-a)$

Write an equation for the tangent line to the function at the given point in point-slope form:

1. $f(x)=x^2+5x-3 \quad (-1,-7)$

$f'(x)=2x+5$

$f'(-1)=2(-1)+5=3$ ↳ slope of tangent line

Tangent line:

$y+7=3(x+1)$

2. $f(x)=\sqrt{x+3} \quad (6,3)$

$f(x)=(x+3)^{1/2}$

$f'(x)=\frac{1}{2}(x+3)^{-1/2}=\frac{1}{2\sqrt{x+3}}$

$f'(6)=\frac{1}{2\sqrt{6+3}}=\frac{1}{6}$ → slope of tangent line

Tangent line:

$y-3=\frac{1}{6}(x-6)$

3. $f(x)=3x^2(x+2)^2 \quad (1,27)$

$f'(x)=3x^2\cdot 2(x+2)^{1}(1)+(x+2)^2\cdot 6x$ (product rule)

$f'(1)=3(1)^2\cdot 2(1+2)(1)+(1+2)^2\cdot 6(1)$

$f'(1)=72$ → slope of tangent line

Tangent line:

$y-27=72(x-1)$

4. $f(x)=\frac{1}{\sqrt{x}} \quad \left(4,\frac{1}{2}\right)$

$f(x)=x^{-1/2}$

$f'(x)=-\frac{1}{2}x^{-3/2}=\frac{-1}{2\sqrt{x^3}}$

$f'(4)=\frac{-1}{2\sqrt{4^3}}=\frac{-1}{16}$ → slope of tangent line

Tangent line

$y-\frac{1}{2}=\frac{-1}{16}(x-4)$